



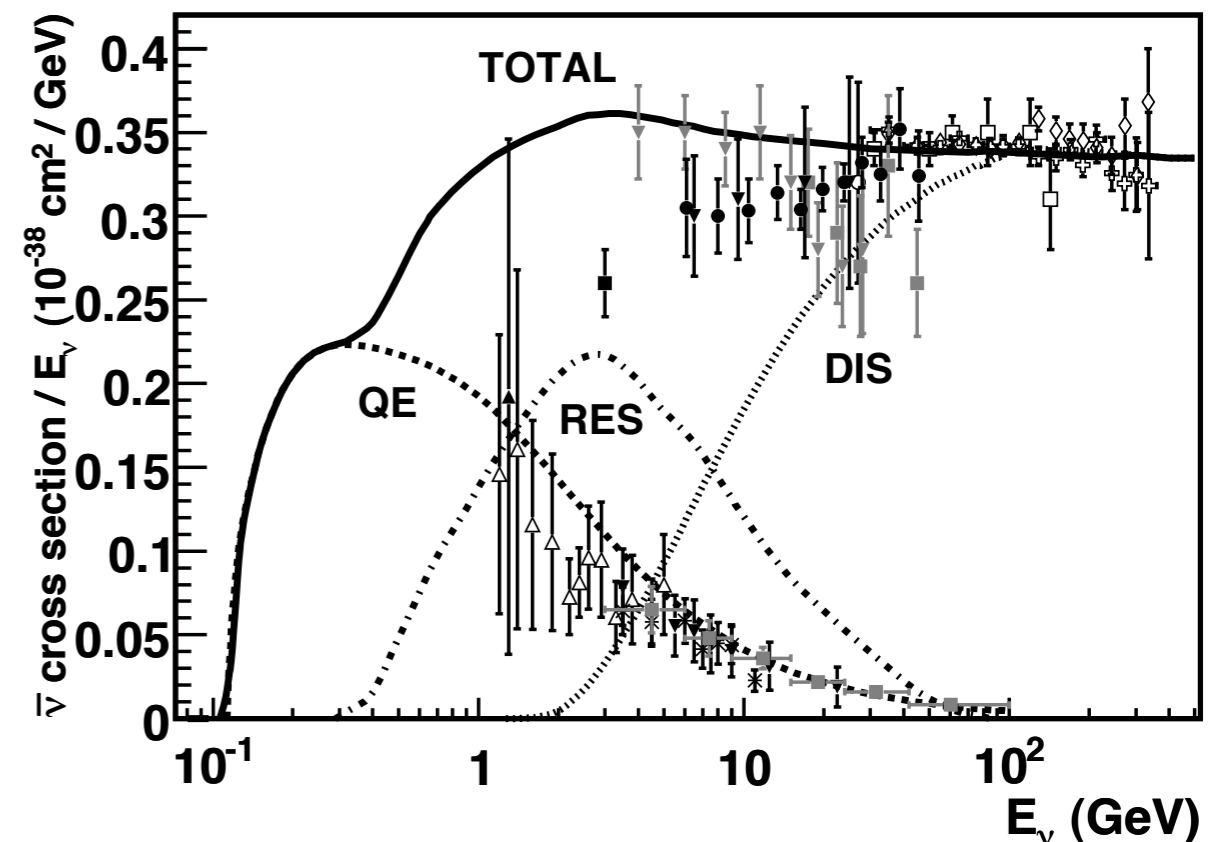
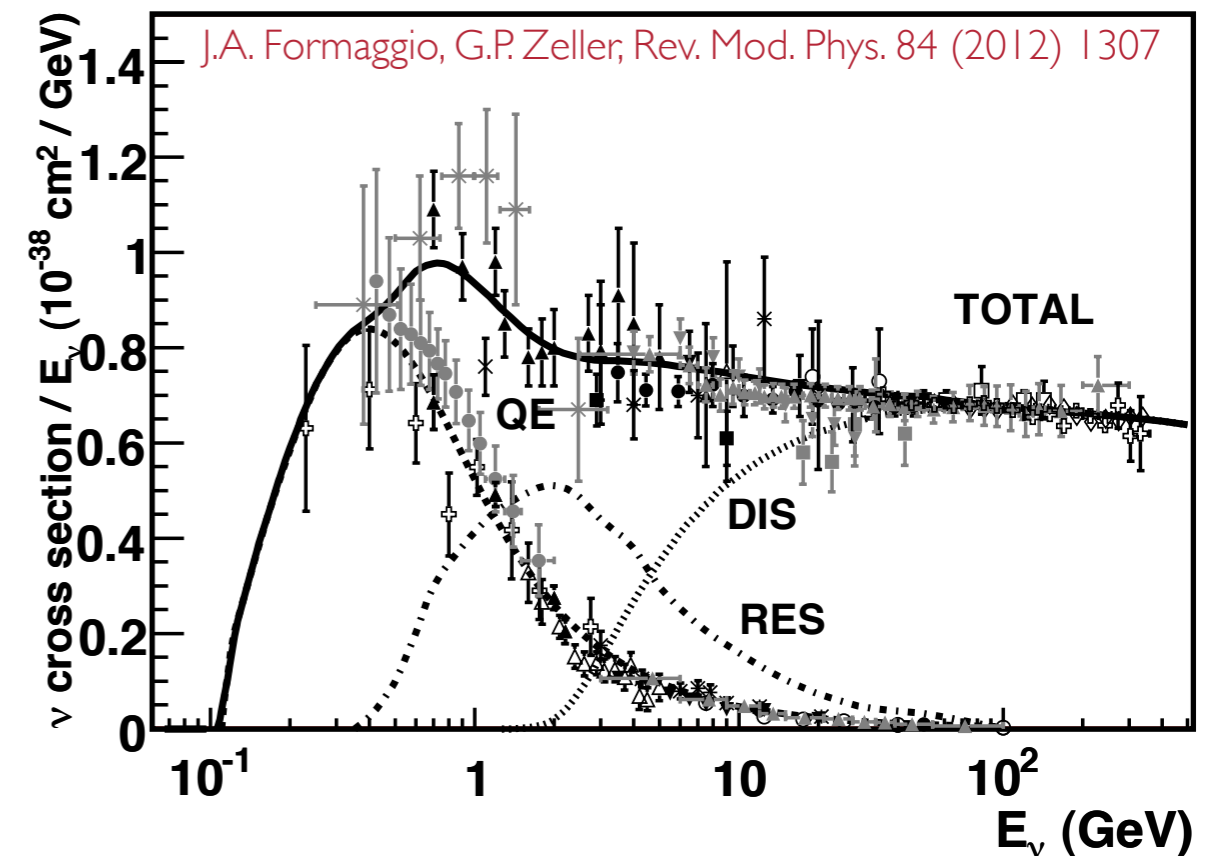
Lattice QCD for Neutrino Physics

William Detmold
MIT

- Neutrino interactions with nucleons and nuclei
- Nucleon and transition form factors
- Deep inelastic structure
- Nuclear effects

Neutrino Scattering

- $\nu N, \bar{\nu} A$ cross sections needed to extract oscillation parameters
- Scatter with various kinematics (beams are not mono-energetic)
- (Quasi)-elastic scattering determined by nucleon axial, PS, ... form factors
- Resonance: determined by transition FFs
- DIS: parton distributions



Neutrino Scattering

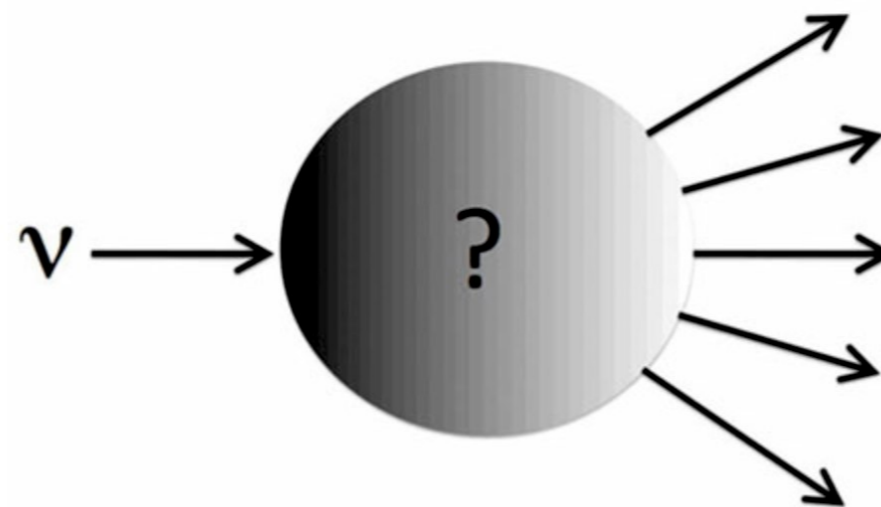
- $\nu N, \bar{\nu} A$ cross sections needed to extract of oscillation parameters

- Scatter (beams

- (Quasi) determ ... form

- Resona transiti

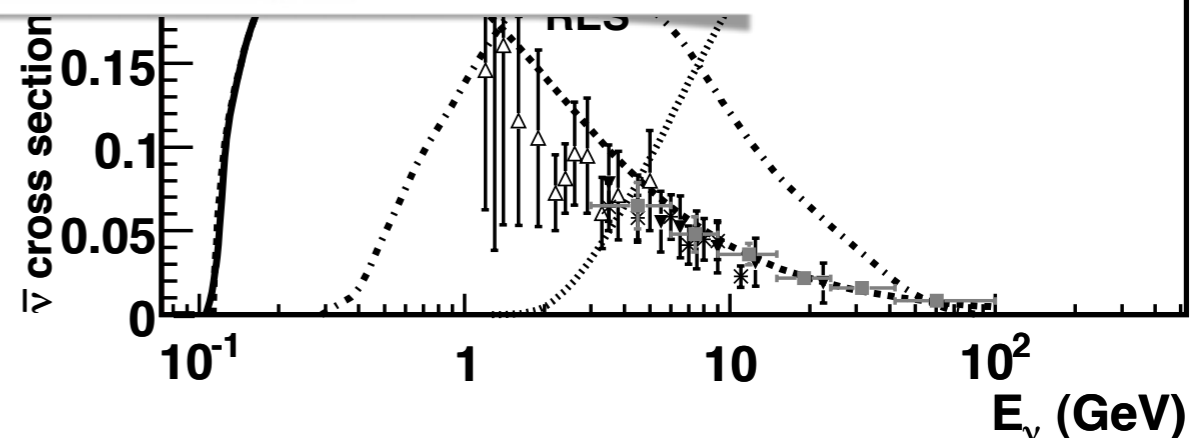
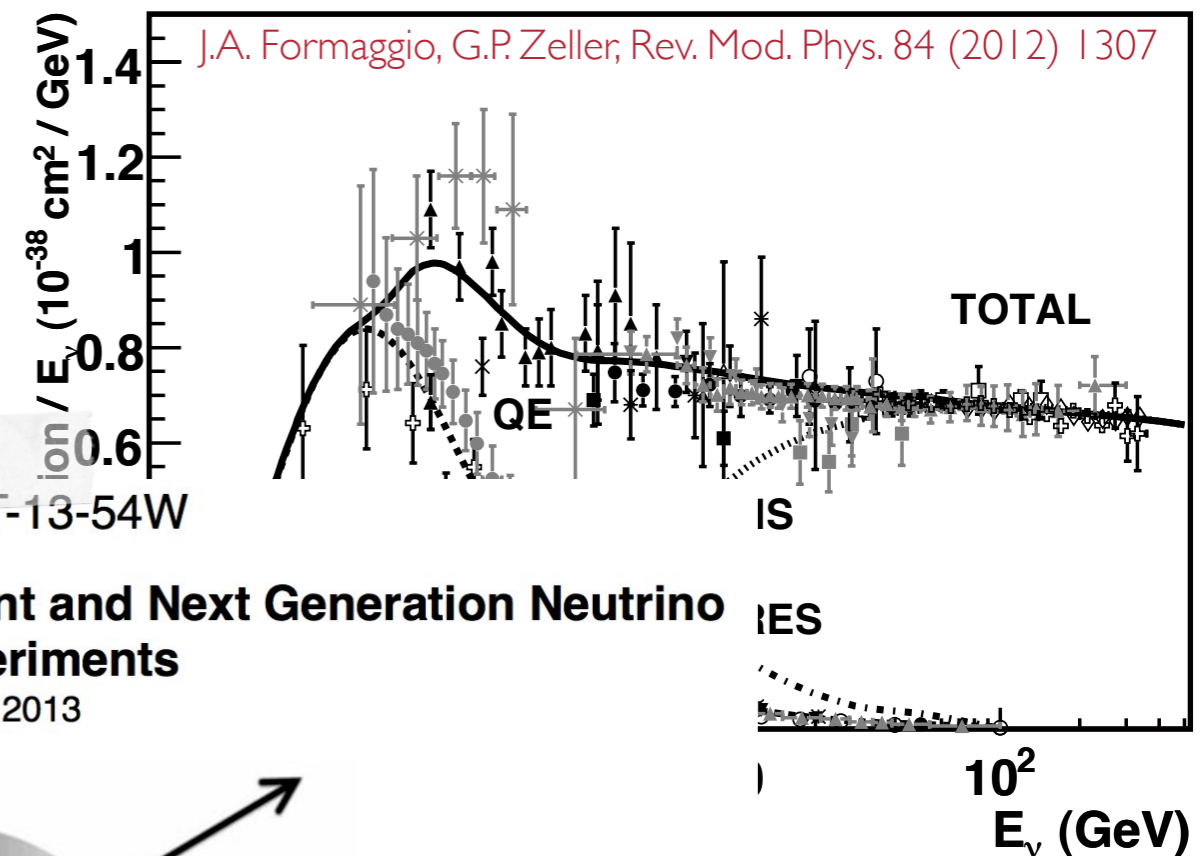
- DIS: parton distributions



INT Workshop INT-13-54W

Neutrino-Nucleus Interactions for Current and Next Generation Neutrino Oscillation Experiments

December 3-13, 2013



(Quasi-)elastic scattering

- Elastic scattering determined by nucleon FFs

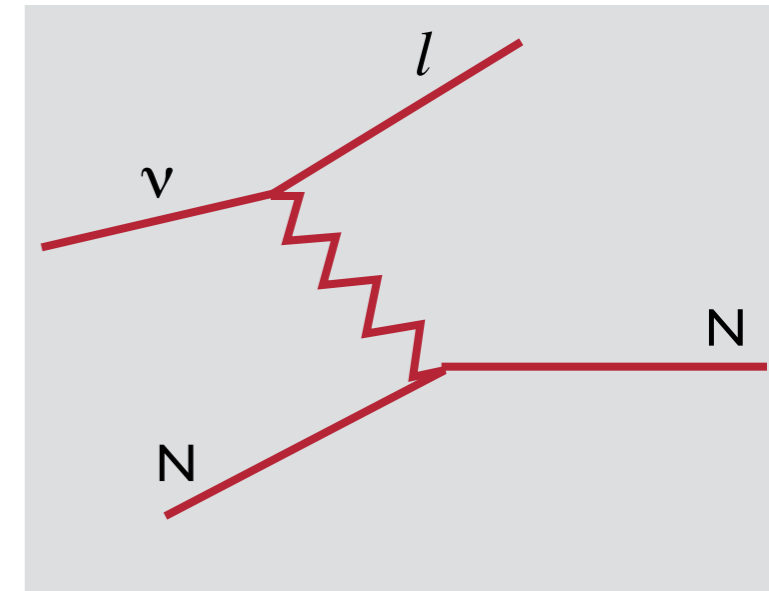
$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 M^2 |V_{ud}|^2}{8\pi E_\nu^2} \left[A \pm \frac{(s-u)}{M^2} B + \frac{(s-u)^2}{M^4} C \right]$$

$$A = \frac{(m^2 + Q^2)}{M^2} \left[(1 + \eta) F_A^2 - (1 - \eta) F_1^2 + \eta(1 - \eta) F_2^2 + 4\eta F_1 F_2 - \frac{m^2}{4M^2} ((F_1 + F_2)^2 + (F_A + 2F_P)^2 - \left(\frac{Q^2}{M^2} + 4 \right) F_P^2) \right]$$

$$B = \frac{Q^2}{M^2} F_A (F_1 + F_2)$$

$$C = \frac{1}{4} (F_A^2 + F_1^2 + \eta F_2^2)$$

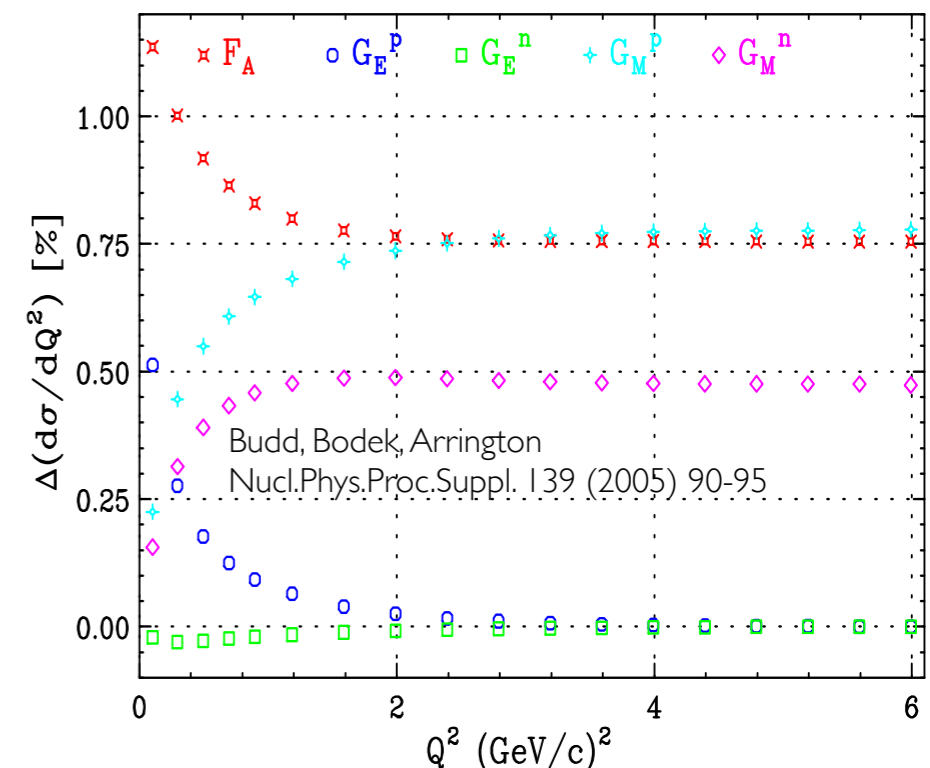
- Vector current: $F_{1,2}$ FFs well constrained from e -N scattering
- Axial vector current: $F_{A,P}$ axial vector and induced pseudo-scalar FFs
 - Much less known experimentally (ν -d scattering and pion electro-production, g_P from muon capture)



$$E_\nu = \frac{m_\mu^2 - (m_p - E_b)^2 - m_\mu^2 + 2(m_p - E_b)E_\mu}{2(m_p - E_b - E_\mu + p_\mu \cos \theta_\mu)}$$

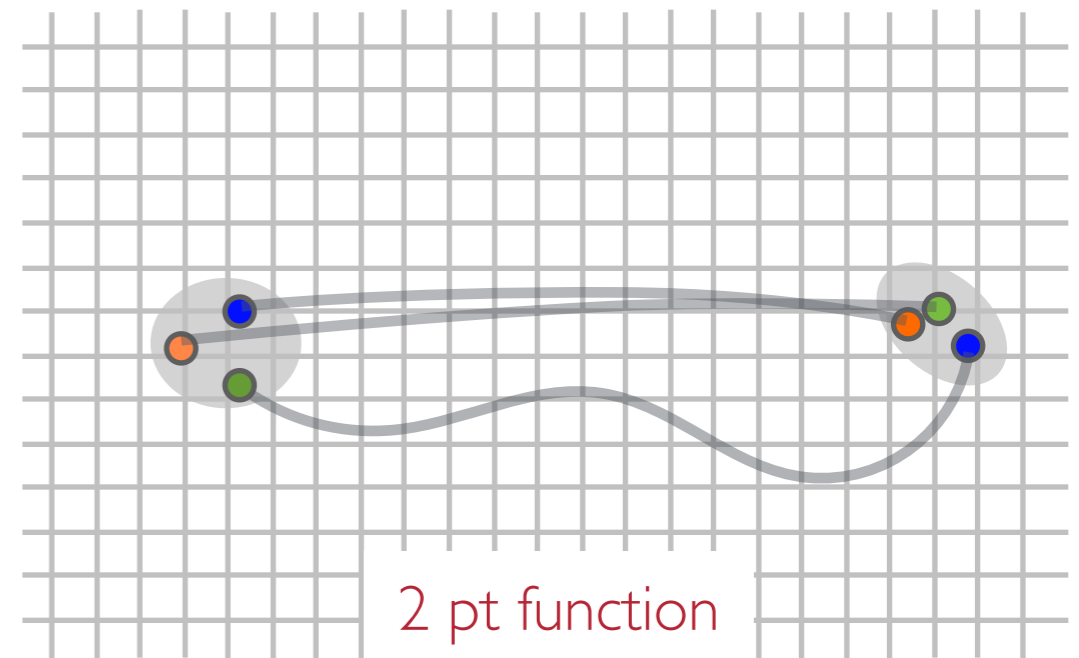
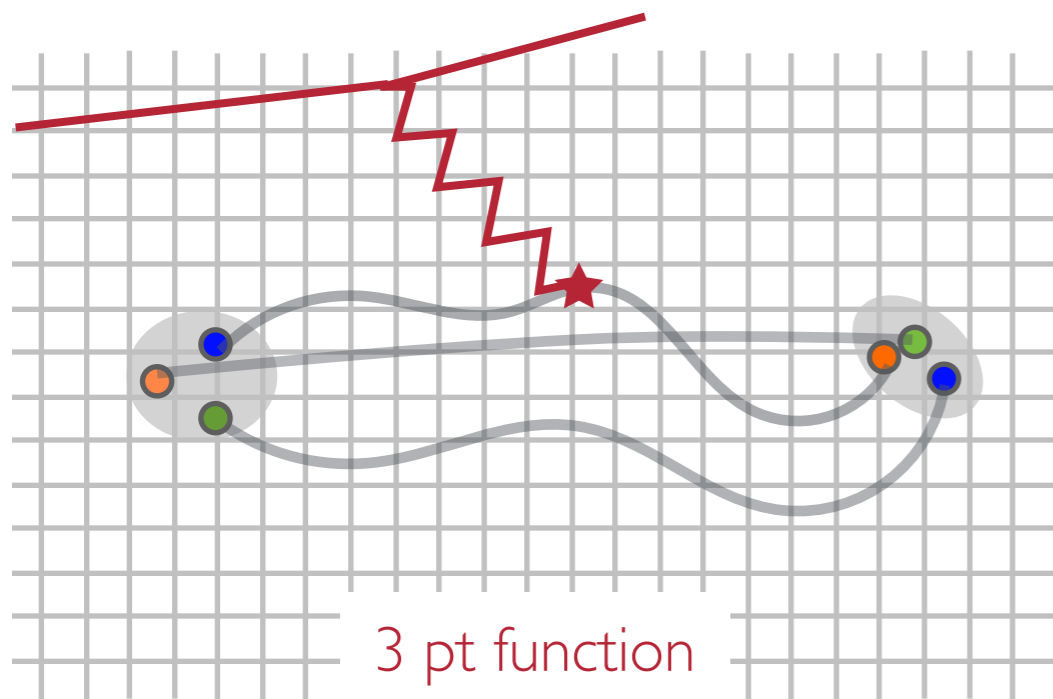
$$Q^2 = 2E_\nu(E_\mu - p_\mu \cos \theta_\mu) - m_\mu^2,$$

QE, ν_μ , $\Delta(d\sigma/dQ^2)$ [%] for 1% Change in FF, $M_A=1$



Nucleon form factors

- LQCD FFs studied from ratios of 2- and 3-point correlators
[Martinelli & Sachrajda 1988]



$$R(t, \tau; \mathbf{q}) \sim \sum_{\mathbf{y}} e^{i\mathbf{q} \cdot \mathbf{y}} \frac{\langle 0 | \chi(0) \mathcal{O}(\mathbf{y}, \tau) \chi^\dagger(t) | 0 \rangle}{\langle 0 | \chi(0) \chi^\dagger(t) | 0 \rangle} \longrightarrow \langle N(p) | \mathcal{O} | N(p + q) \rangle$$

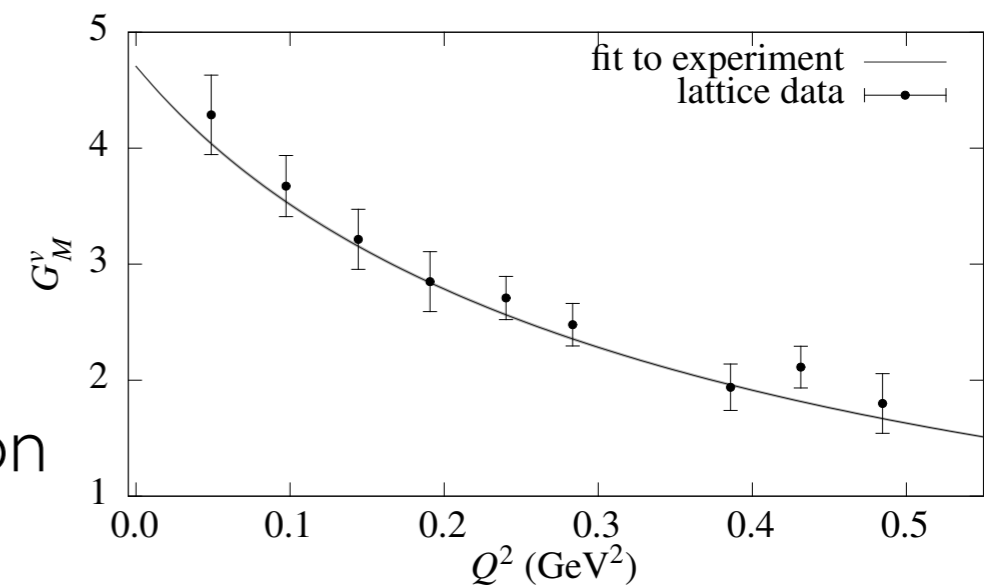
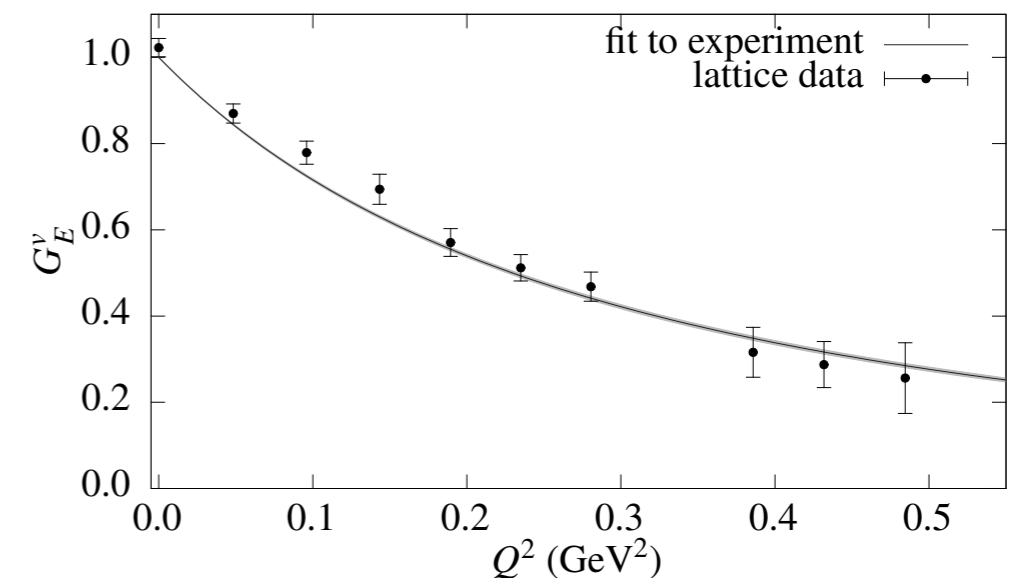
- Determines ground state FF at large source/operator/sink separation

Isovector nucleon form factors

- EM FFs studied by many groups
- Recently: almost-physical quark mass
- Quite different at heavy masses
- Nucleon axial and PS FFs accessible with same techniques
- Statistical uncertainties
- Systematics: volume, temporal extent, lattice spacing, quark mass, $N_f=2+1$
- AV current not conserved: renormalisation
- Large Q^2 : challenging in large volumes
[Hsu/Fleming; Renner; Cohen/Lin, Della Morte et al.; Roberts et al.]

J. Green et al. (LHPC), arXiv:1310.7043

$m_\pi = 149$ MeV



Isovector axial and PS form factors

	N	Action	a [fm]	m	L [fm]	T [fm]	m	T	Renorm.	Q	Ref
ETMC	2	Twisted Mass	0.06–0.09 (3)	260–470	2.1–2.8	4–6	3.3–5	1.0–1.1 (2)	Nonpert	0–1.5	PRD 2011
ETMC	2+1+1	Twisted Mass	0.07, 0.09	210, 350	2.6, 3.1	5, 6	3.3	1.0, 1.2	Nonpert	0–0.7	PRD 2013
ETMC	2+1+1	Twisted Mass	0.08	380	2.5	5.0	4.9	0.9–1.6 (9)	Nonpert	0	PLB 2011
PNDME	2+1+1	Clover on HISQ	0.12	220, 300	2.9, 3.8	7.6	4.4, 4.6	1.0–1.4 (5)	Nonpert	0, in progress	1306.5435
LHPC	2+1	Clover	0.11	320	3.6	10	6	0.7–1.6	None	0–0.7	Green thesis
LHPC	2+1	Clover	0.09, 0.11	150–350	2.8–5.6	2.8–5.6	3.6–5	0.9–1.4	Nonpert	0–0.7	Green ongoing
LHPC	2+1	DWF on Asqtad	0.12	300–750	2.5, 3.5	7.7	3.7–9	1.1	Nonpert	0–0.6	PRD 2010
LHPC	2+1	DWF	0.08, 0.11	330–400	2.7	5.4	4–5.5	1.0, 1.2	Nonpert	0	PRD 2010
CLS/M	2	Clover	0.05–0.08	195–650	2.0–4.0	4–6	4–8	0.8–1.3 (4)	Nonpert	0	PRD 2012
QCDSF	2	Clover	0.06–0.08	130–500	1.0–3.5	2.4–4.8	2.6–9	~1.1 +	Nonpert Improved	0	1302.2233
RBC	2+1	DWF	0.14	170, 250	4.6	9.2	3.9, 5.8	1.0, 1.3	Nonpert	0	LATT13
RBC	2+1	DWF	0.11	350–700	1.8, 2.7	5.4	4–8	1.4	Nonpert	0–0.75	PRD 2009
CSSM	2+1	Clover	0.09	290	2.9	5.8	4.3	0.5 (var)	Nonpert	0	PLB 2013
Cohen/Lin	2+1	Aniso Clover	0.12	450–870	2.0	4.4	4.5–8	variational	Perturb	0–4.0	Latt2010

Recent unquenched studies of axial vector current form factors/axial charge with results on arXiv
 See P Hägler, Phys. Rep. 490 (2010), 49 for earlier works

Isovector axial and PS form factors

My assessment of current calculations: which systematics are treated well and which could be improved

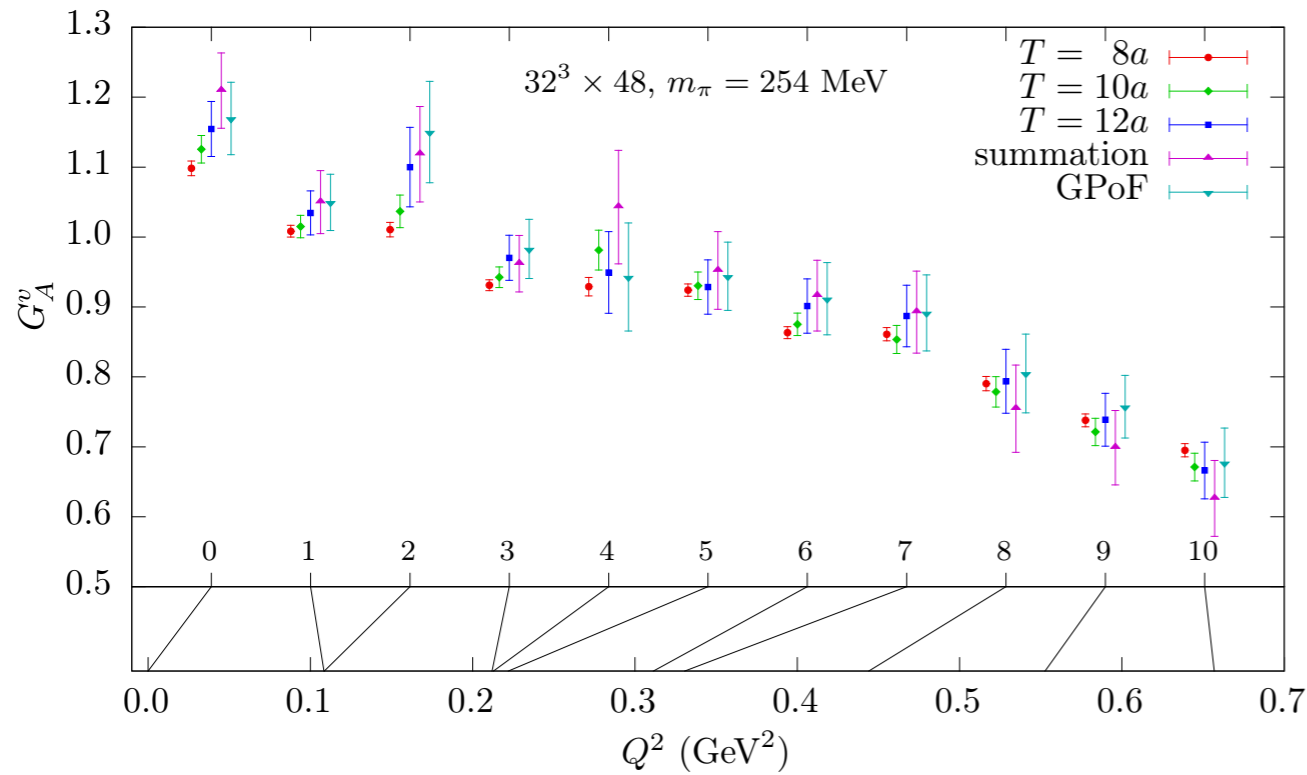
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Recent unquenched studies of axial vector current form factors/axial charge with results on arXiv

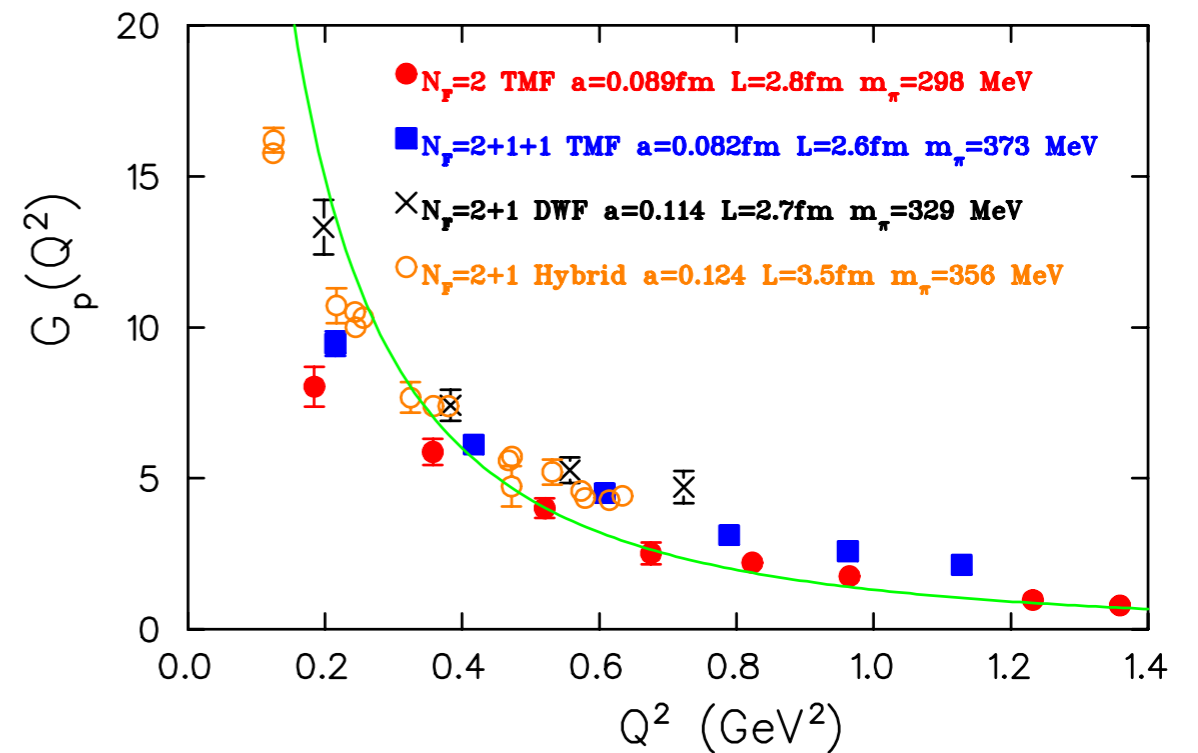
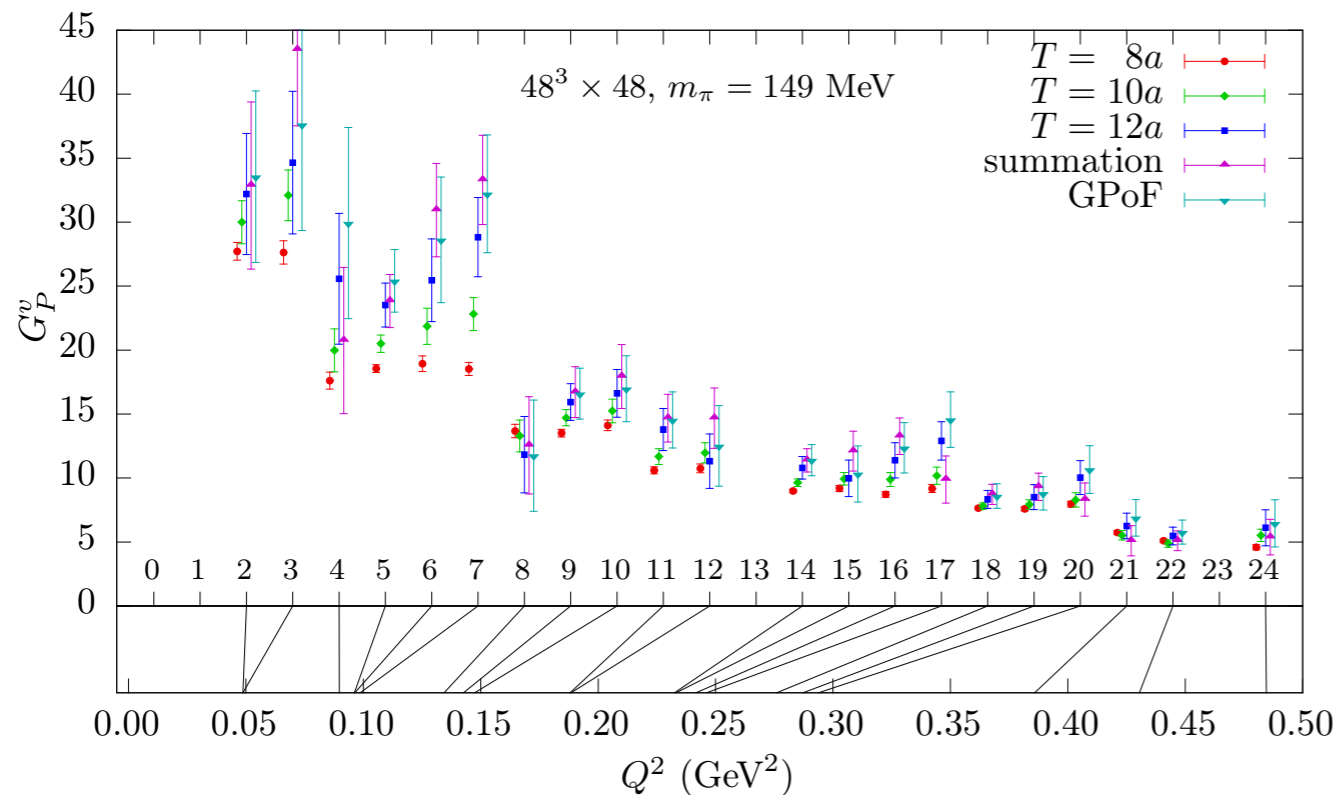
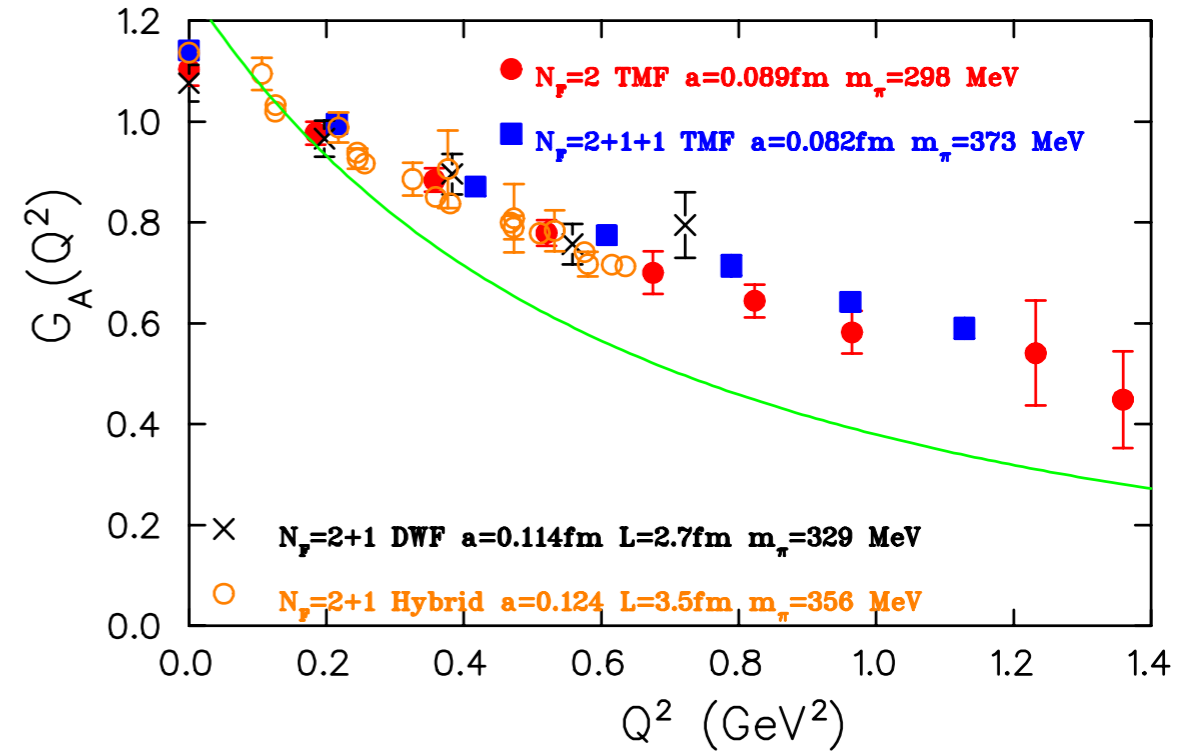
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Isovector axial and PS form factors

J Green thesis (LHPC), 2013

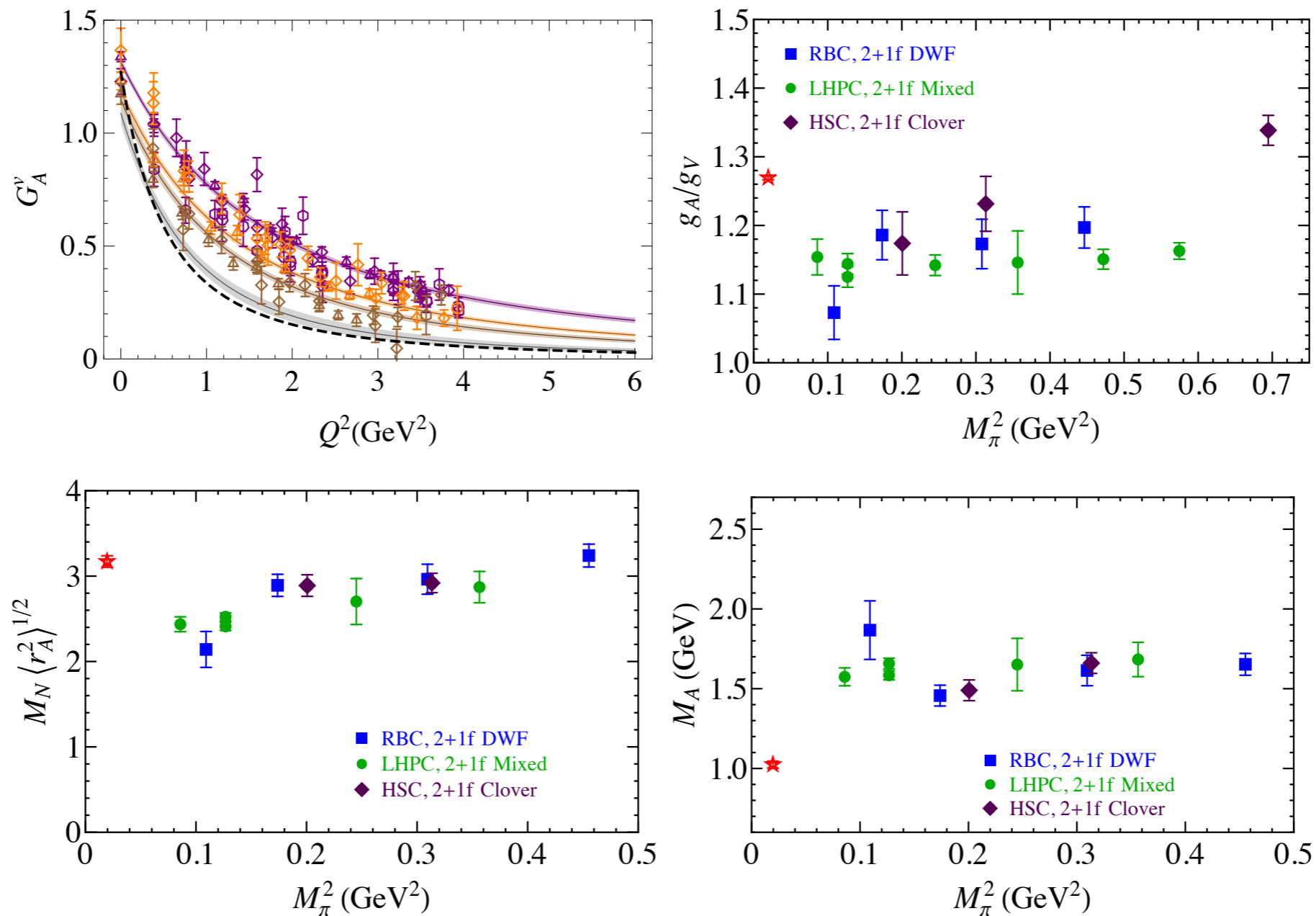


C Alexandrou et al., (ETMC) Phys.Rev. D88 (2013) 014509



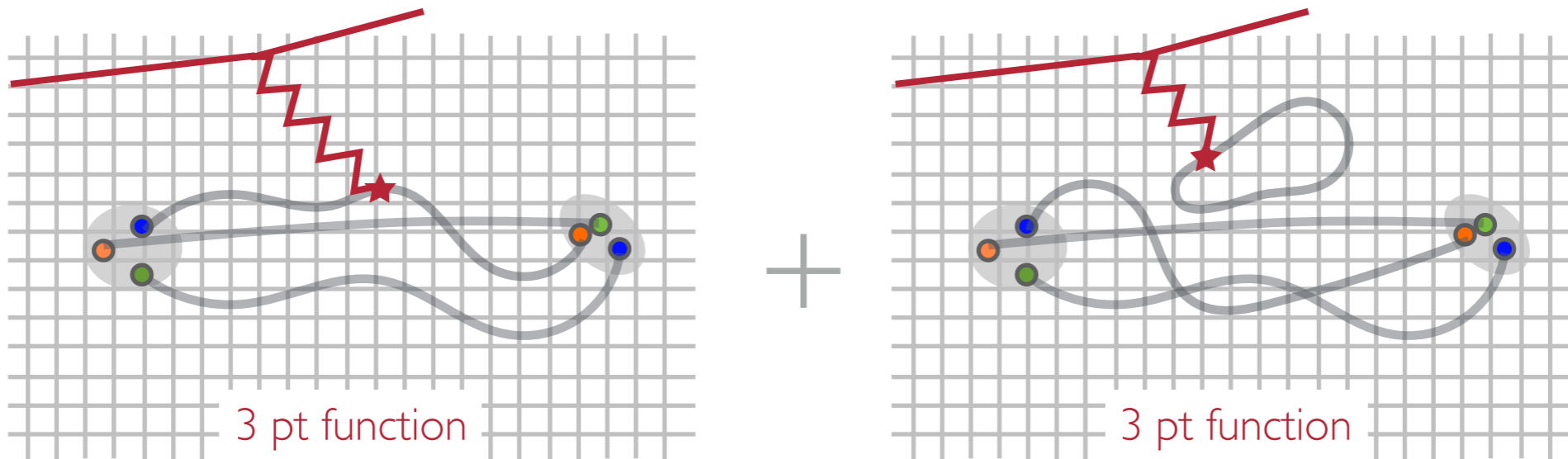
Isovector axial mass, radius

- Perform dipole fits to extract axial mass and radius
- Dipole fails to describe experiment in vector FF case



Isoscalar nucleon form factors

- Isoscalar FFs more difficult: “quark-line disconnected” contributions

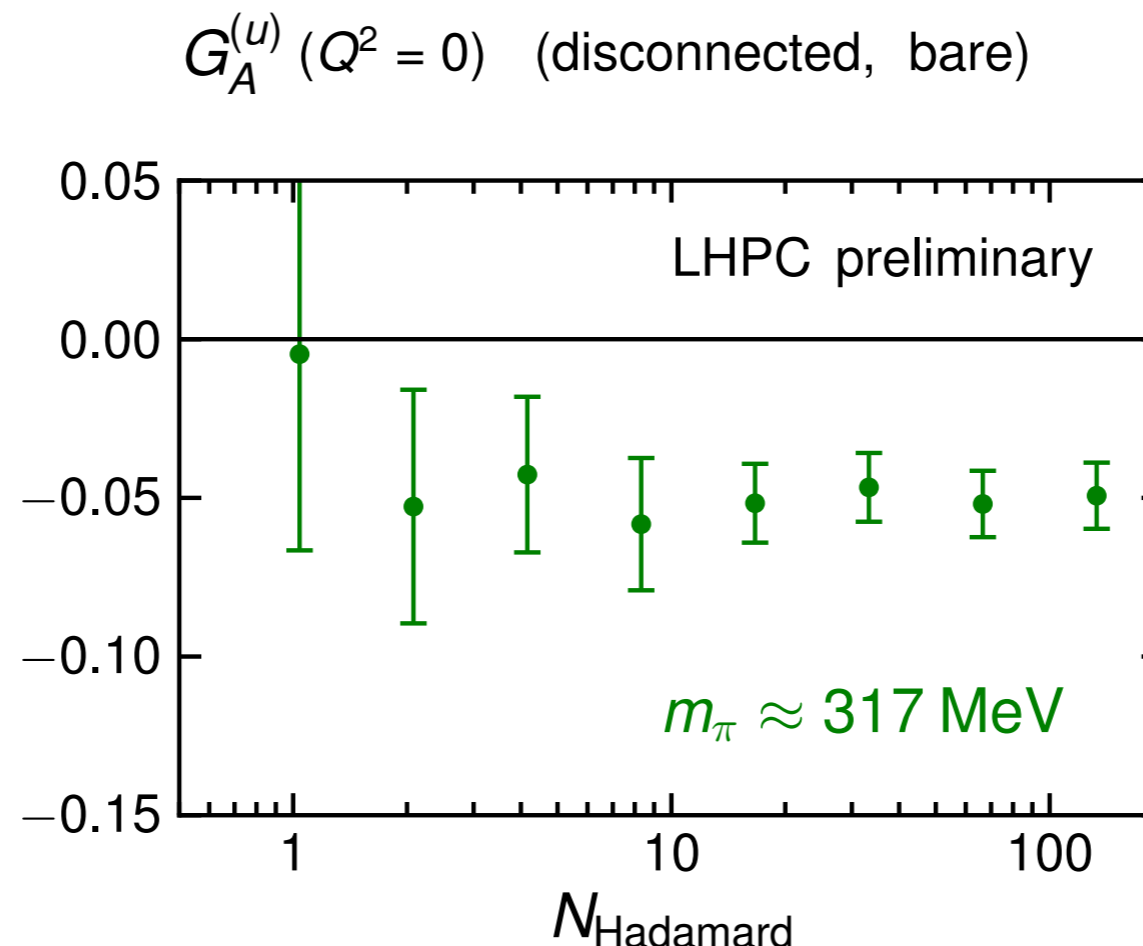


- Various techniques for stochastic estimation
- Disconnected contributions typically small
- Strange quark v/av FFs relevant for NC scattering
 - Also difficult experimentally
 - Important place for lattice contributions

Isoscalar axial and pseudo-scalar FFs

[illegible]

- Truncated solver methods, all mode averaging appear effective
- Promising new way of stochastically sampling: “hierarchical probing” [Stathopoulos, Laeuchli & Orginos]
- Being used by Meinel et al.



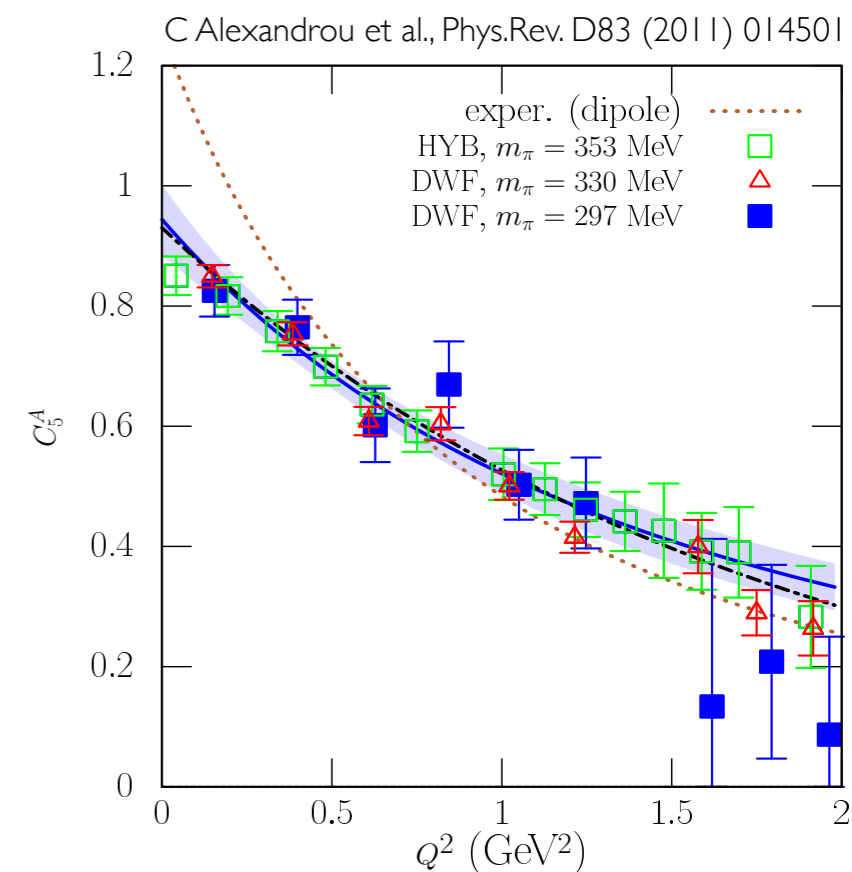
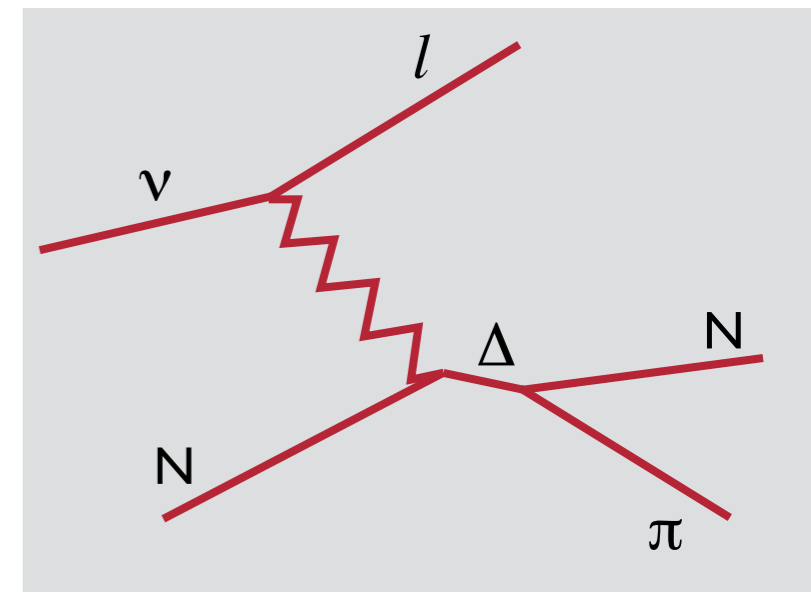
Transition form factors

- Resonance region
- Dominant contribution from Δ resonance but N^* 's also important at high E_ν

$$\langle \Delta(p', s') | A_\mu^3 | N(p, s) \rangle = i \sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}') E_N(\mathbf{p})} \right)^{1/2} \bar{u}_{\Delta+}^\lambda(p', s')$$

$$\left[\left(\frac{C_3^A(q^2)}{m_N} \gamma^\nu + \frac{C_4^A(q^2)}{m_N^2} p'^\nu \right) (g_{\lambda\mu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu}) q^\rho + C_5^A(q^2) g_{\lambda\mu} + \frac{C_6^A(q^2)}{m_N^2} q_\lambda q_\mu \right] u_P(p, s)$$

- Very difficult to access experimentally
Guess from PCAC
- QCD calculations possible
- Single calculation to date [Alexandrou et al.]
- Need to account for unstable nature of resonance: extract $N \rightarrow N\pi$ transition FFs

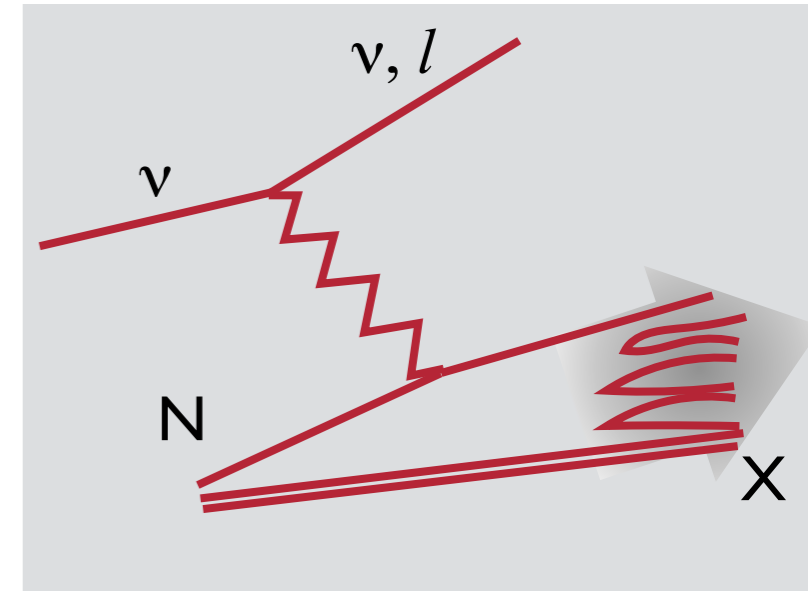


- In inelastic regime, quark PDFs of nucleon control scattering x-sec

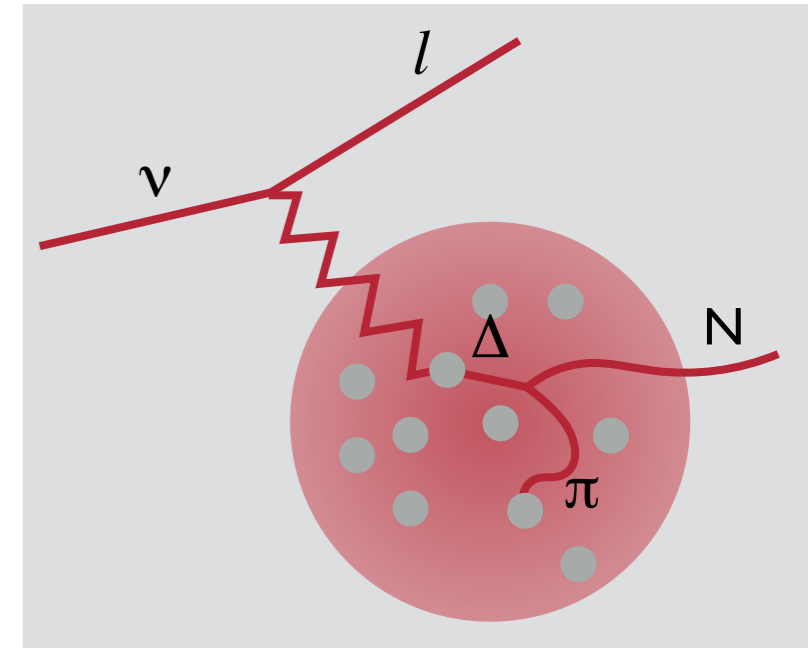
$$\frac{d^2 \sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G_F^2 M E_\nu}{\pi (1 + Q^2/M_{W,Z}^2)^2} \left[\frac{y^2}{2} 2x F_1(x, Q^2) + \left(1 - y - \frac{Mxy}{2E}\right) F_2(x, Q^2) \right. \\ \left. \pm y \left(1 - \frac{y}{2}\right) x F_3(x, Q^2) \right]$$

- Both CC and NC processes relevant
- Known from global analyses to sufficient(?) accuracy
 - Nuclear effects may be different in νA vs. eA (MINER νA)
- LQCD typically* calculates low moments of PDFs

$$\langle N | \bar{q} \gamma_{\{\mu_1} D_{\mu_2} \dots D_{\mu_n\}} q | N \rangle$$
- Clean access to strangeness
 - Useful contributions to neutrino program??

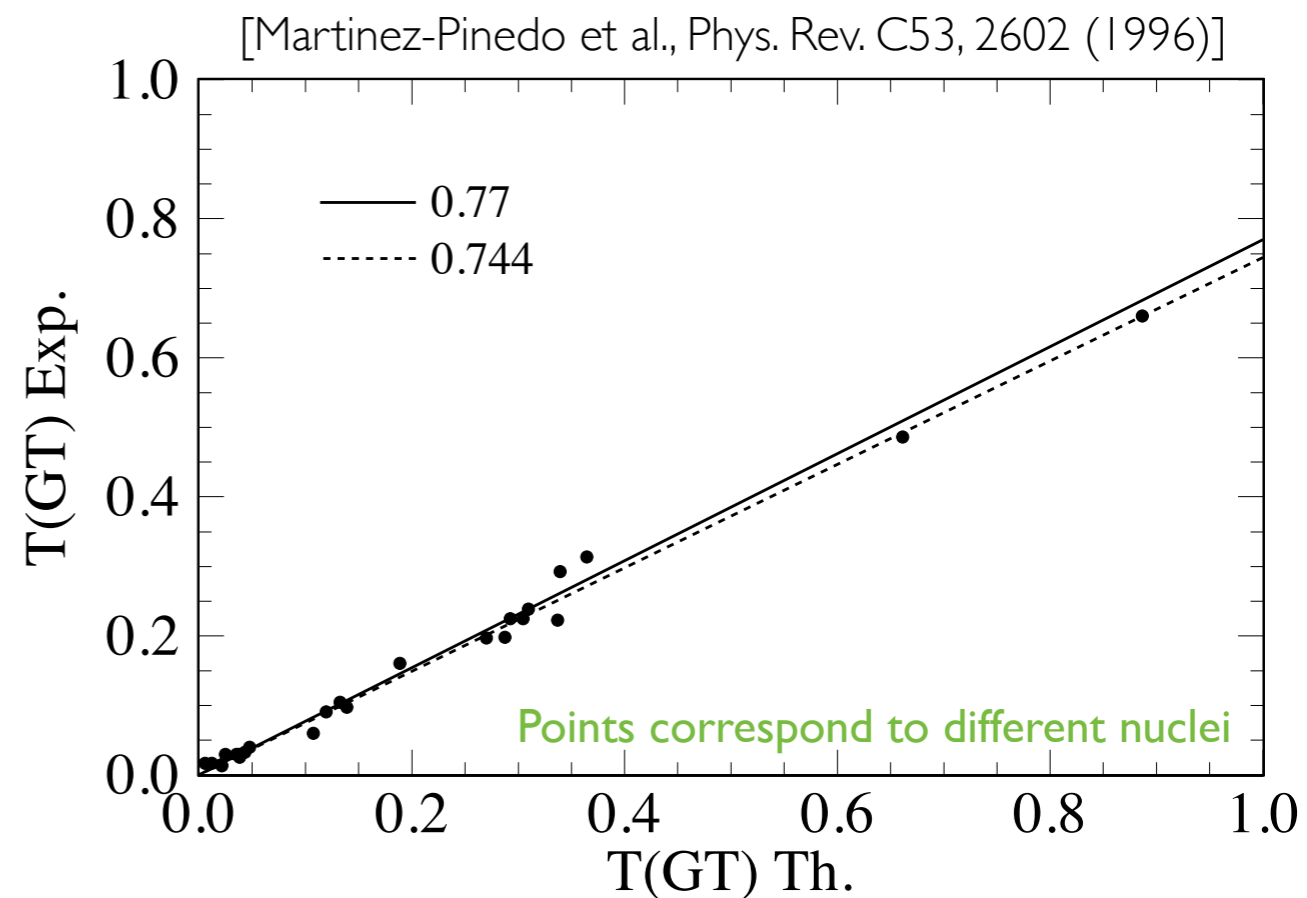


- Targets are nuclei (C, Fe, Ar, Pb, CH_x, H₂O) so how relevant are nucleon FFs, PDFs?
- EMC effect
- Quenching of g_A in GT transitions
- Estimated $\sim 10\%$ effects on oscillation parameters [C Mariani, INT]
- Experimental investigations: MINERvA
- QCD calculations of few nucleon observables input for EFT-based few-body methods



Gamow-Teller: axial charge in nuclei

- Gamow-Teller transitions in nuclei are a stark example of problems
- Well measured
- Best nuclear structure calculations are systematically off by 20–30%
- Large range of nuclei ($30 < A < 60$) where spectrum is well described
- QRPA, shell-model,...
- Correct for it by “quenching” axial charge in nuclei ...

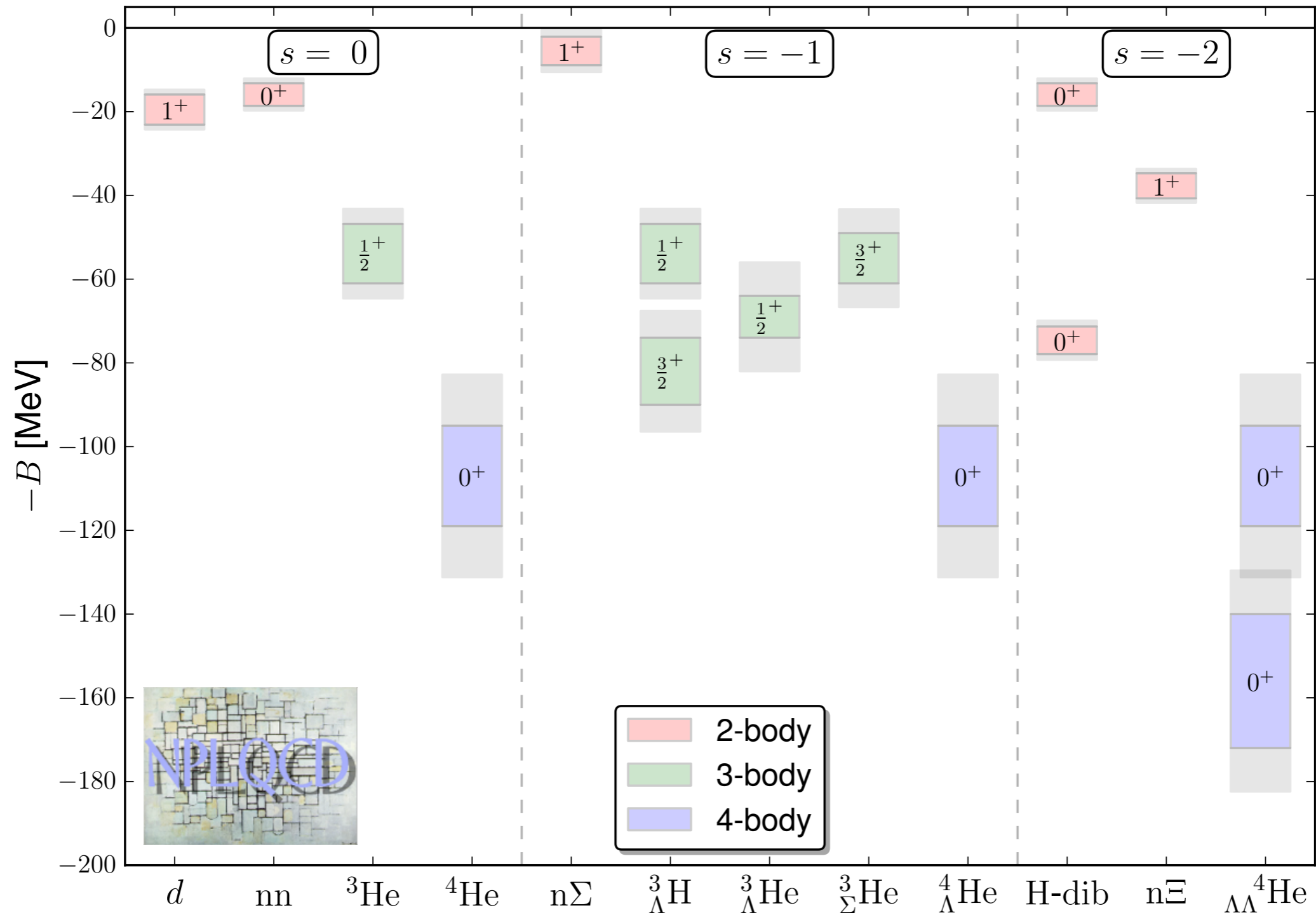


$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \rightarrow f}}$$

$$\langle \boldsymbol{\sigma} \boldsymbol{\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_{\pm}^k || i \rangle}{\sqrt{2J_i + 1}}$$

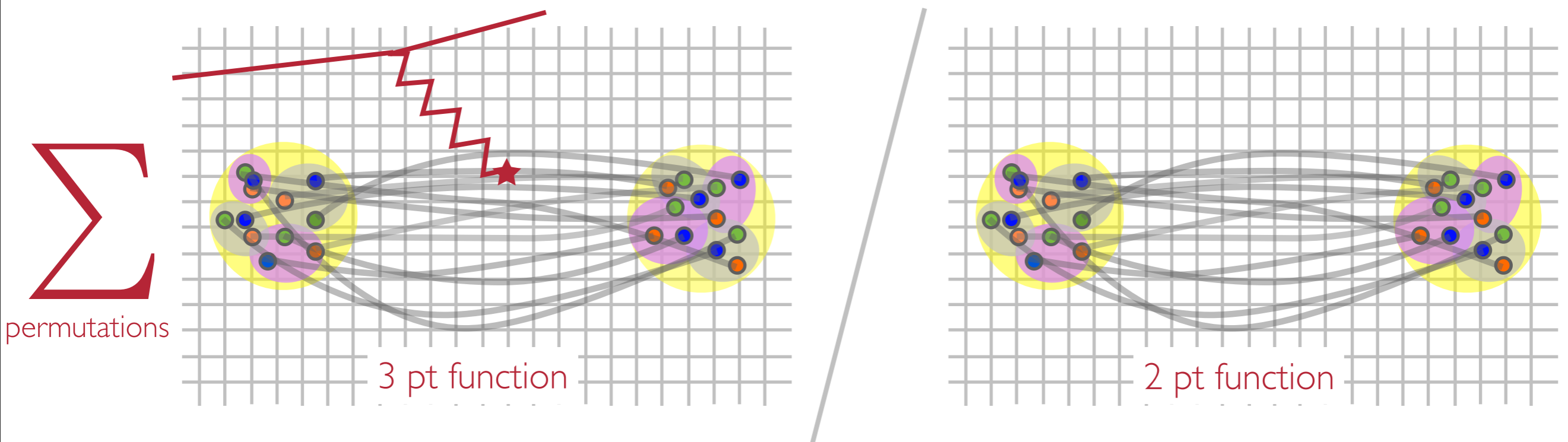
Nuclei in QCD ($A=2,3,4$)

@ $m_\pi = 800$ MeV



Nuclear matrix elements

- Calculations of matrix elements of currents in light nuclei just beginning
- For deeply bound nuclei, use the same techniques as for single hadron matrix elements



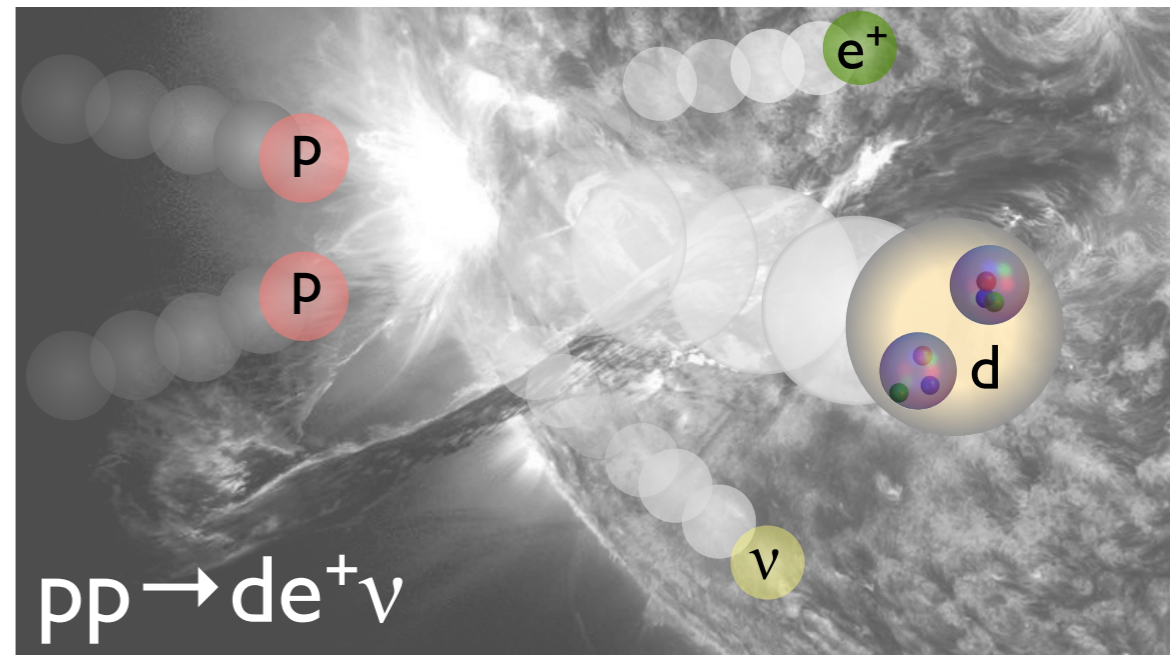
- For near threshold states, need to be careful with volume effects

- Power counting of nuclear effective field theory:
 - 1-body currents are dominant
 - 2-body currents are sub-leading but non-negligible and higher-body currents are even less important
- Determine one body contributions from single nucleon
- Determine few-body contributions from $A=2,3,4\dots$
- Match EFT and many body methods to LQCD to extend to larger nuclei

- Axial coupling to NN system
 - pp fusion: “Calibrate the sun”
 - Muon capture: MuSun @ PSI
 - $d\nu \rightarrow nne^+ : \text{SNO}$
- Twist-2 operators: eg EMC effect

$$\langle N, Z | \bar{q} \gamma_{\{\mu_1} D_{\mu_2} \dots D_{\mu_n\}} q | N, Z \rangle$$

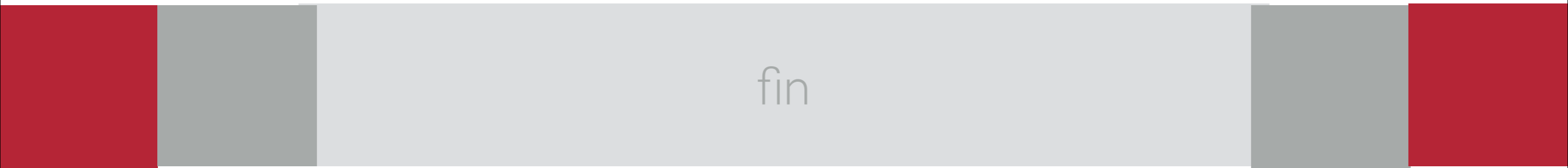
- Proof of principle (moments of pion PDF in pion gas) [WD, HW Lin 1112.5682]



- Lattice efforts have potential to impact ν energy determinations
- Outlook for future calculations
 - Connected FFs at 3% circa 2017 [USQCD whitepaper]
 - Disconnected contributions including strangeness: generally small, so overall 3% only needs $\sim 20\%$ on these - feasible
 - Large momentum FFs ($> \text{GeV}$) difficult in large volumes but less precision needed. Ideas exist, need testing
 - Transition FFs: tools exist for Δ , but developments necessary for higher states above $N\pi\pi$ inelastic threshold
 - 2-, 3- body matrix elements to constrain nuclear effects

Level of uncertainty





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